1. The pitch circles and addendum circles of two gears are shown below.

(a) [10%] Please draw the line of action and indicate pressure angle $\phi$, base circle radii $r_{b2}, r_{b3}$.

Also properly indicate the pitch point $K$, length of action $AB$, and meshing limits $CD$.

(b) [10%] Please derive the formula for the length of action $AB$.

\[
Z = r_{AB} = r_{AK} + r_{KB} \quad 2\%
\]

From triangle $O_3AD$ and $O_3KD$

\[
r_{AD} = \left[ (r_3 + a)^2 - (r_{b3})^2 \right]^\frac{1}{2} \quad \text{and} \quad r_{KD} = r_3\sin\phi
\]

From triangle $O_2BC$ and $O_2KC$

\[
r_{CB} = \left[ (r_2 + a)^2 - (r_{b2})^2 \right]^\frac{1}{2} \quad \text{and} \quad r_{CK} = r_2\sin\phi \quad 2\%
\]

\[
Z = r_{AB} = r_{AK} + r_{KB} = r_{AD} + r_{CB} - (r_{CK} + r_{KD})
\]

\[
= \left[ (r_3 + a)^2 - (r_{b3})^2 \right]^\frac{1}{2} + \left[ (r_2 + a)^2 - (r_{b2})^2 \right]^\frac{1}{2} - (r_2 + r_3)\sin\phi \quad 6\%
\]
let the \( r_{b3} = r_3 \cos \phi \); \( r_{b2} = r_2 \cos \phi \)

\[
Z = \left[ (r_3 + a)^2 - (r_3 \cos \phi)^2 \right]^{1/2} + \left[ (r_2 + a)^2 - (r_2 \cos \phi)^2 \right]^{1/2} - (r_2 + r_3) \sin \phi
\]

or

let the \( r_{3} \sin \phi = \left[ r_3^2 - (r_{b3})^2 \right]^{1/2} \); \( r_{2} \sin \phi = \left[ r_2^2 - (r_{b2})^2 \right]^{1/2} \)

\[
Z = \left[ (r_3 + a)^2 - (r_{b3})^2 \right]^{1/2} + \left[ (r_2 + a)^2 - (r_{b2})^2 \right]^{1/2} - \left( \left[ r_3^2 - (r_{b3})^2 \right]^{1/2} + \left[ r_2^2 - (r_{b2})^2 \right]^{1/2} \right)
\]
2. The following table provides information for the reverted gear train shown. All gears were manufactured using a hob cutter with 25° pressure angle, 0.375 inch hob pitch, and full depth (addendum and dedendum are 1.0/P and 1.25/P, respectively).

<table>
<thead>
<tr>
<th>Gear No.</th>
<th>Straight spur</th>
<th>Helical spur</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of teeth</td>
<td>2 10</td>
<td>3 70</td>
</tr>
</tbody>
</table>

Determine
(a) [5%] the base circle radii of gears 2 and 3
(b) [5%] the length of action of gears 2 and 3
(c) [5%] the distances between the meshing limit points of gears 2 and 3
(d) [5%] whether interference will occur between gears 2 and 3
(e) [5%] the helix angle of gear 5
(f) [5%] the addendum circle diameter of gear 4

(a)
\[
r_2 = \frac{N_{P_c}}{2\pi} = \frac{10 \times 0.375}{2\pi} = 0.597 \text{ in}
\]
\[
r_3 = \frac{N_{P_c}}{2\pi} = \frac{70 \times 0.375}{2\pi} = 4.178 \text{ in}
\]
\[
r_{b_2} = r_2 \cos(25°) = 0.541 \text{ in} \quad 5%
\]
\[
r_{b_3} = r_3 \cos(25°) = 3.787 \text{ in} \quad 5%
\]

(b)
Addendum: \( a = \frac{1}{10/(2 \times 0.597)} = 0.119 \text{ in} \quad 2\%
\]
\[
Z = \left[ (r_3 + a)^2 - (r_{b_3})^2 \right]^{\frac{1}{2}} + \left[ (r_2 + a)^2 - (r_{b_2})^2 \right]^{\frac{1}{2}} - (r_2 + r_3) \sin\phi
\]
\[
= \left[ (4.178 + 0.119)^2 - (3.787)^2 \right]^{\frac{1}{2}} + \left[ (0.597 + 0.119)^2 - (0.541)^2 \right]^{\frac{1}{2}} - (0.597 + 4.178) \times \sin(25°)
\]
\[
= 0.482 \text{ in} \quad 3\%
\]
\[(r_2 + r_3) \times \sin(\phi) = (0.597 + 4.178) \times \sin(25^\circ) = 2.018 \text{ in} \quad 5\%\]

\[
d_{AK} = r_{AD} - r_{KD} = \left[\left(r_3 + a\right)^2 - \left(r_{03}\right)^2\right]^{\frac{1}{2}} - \left(r_3\right) \sin\phi = 0.265 \text{ in}
\]
\[
d_{CK} = \left(r_2\right) \sin\phi = 0.597 \times \sin(25^\circ) = 0.252 \text{ in}
\]
\[
d_{AK} > d_{CK}\]

YES. Since the length of \(d_{AK}\) is larger than the length of \(d_{CK}\). 5%

\[
\therefore \quad \text{This is a reverted gear train}
\]
\[
\therefore c = \frac{d_2}{2} + \frac{d_3}{2} = \frac{d_4}{2} + \frac{d_5}{2} = \frac{1}{2} (m_4 N_4 + m_5 N_5)
\]

The relationship between two helical gears in mesh is \(m_4 = m_5 = m\)

Therefore, \(c = \frac{m}{2} (30 + 48) = 4.775 \text{ in}, m = 0.122\)

\[
p_{d4} = m_4 \pi = 0.122 \times \pi = 0.383 \text{ in}
\]
\[
\psi = \cos^{-1} \left(\frac{0.375}{0.383}\right) = 11.731^\circ \quad 5\%
\]

(f)

Since using the same cutter, the addendum of gear 4 is equal to the addendum of gear 2.
\[
d_4 + 2a = m_4 N_4 + 2 \times 0.119 = 3.66 + 0.238 = 3.898 \text{ in} \quad 5\%
\]