Introduction to Heat and Mass Transfer

Week 5
Example

- Consider an insulated pipeline exposed to atmosphere. Will the critical radius of insulation be greater on calm day as compared to that on windy days?
HW # 2 prob. 2

- Consider an insulated oil pipeline exposed to atmosphere in Alaska. Will the critical radius of insulation be greater in June as compared to that in December?
HW # 2 prob. 3

• A 3 mm diameter and 5 m long electrical wire is wrapped with a 2 mm thick plastic cover, \( k = 0.15 \text{ W/mK} \). Electrical measurements indicate that a current of 10 A passes through the wire and there is a 8 V voltage drop along the wire. The insulated wire is exposed to fluid at 30°C with a heat transfer coefficient of 12 W/m\(^2\)C. For steady operation:

  » Determine the temperature at the interface of the wire and the plastic cover

  » Decide whether doubling the thickness of plastic increases or decreases the heat loss and the interface temperature
Coverage thus far…..

» Steady state conduction through cylinder and sphere

» Thermal conduction resistance of cylinder

\[ R_{t,\text{cond}}^{\text{cylinder}} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi lk} \]

» Thermal conduction resistance of sphere

\[ R_{t,\text{cond}}^{\text{sphere}} = \frac{\left(1/r_i\right) - \left(1/r_o\right)}{4\pi k} \]

» Determination of critical resistance for cylindrical and spherical systems
### Table 3.3  One-dimensional, steady-state solutions to the heat equation with no generation

<table>
<thead>
<tr>
<th></th>
<th>Plane Wall</th>
<th>Cylindrical Wall (^a)</th>
<th>Spherical Wall (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat equation</strong></td>
<td>( \frac{d^2T}{dx^2} = 0 )</td>
<td>( \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0 )</td>
<td>( \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0 )</td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td>( T_{s,1} - \Delta T \frac{x}{L} )</td>
<td>( T_{s,2} + \Delta T \frac{\ln (r/r_2)}{\ln (r_1/r_2)} )</td>
<td>( T_{s,1} - \Delta T \left[ \frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right] )</td>
</tr>
<tr>
<td><strong>Distribution</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Heat flux</strong> ((q))</td>
<td>( k \frac{\Delta T}{L} )</td>
<td>( \frac{k \Delta T}{r \ln (r_2/r_1)} )</td>
<td>( \frac{k \Delta T}{r^2 \left[ (1/r_1) - (1/r_2) \right]} )</td>
</tr>
<tr>
<td><strong>Heat rate</strong> ((q))</td>
<td>( kA \frac{\Delta T}{L} )</td>
<td>( \frac{2\pi Lk \Delta T}{\ln (r_2/r_1)} )</td>
<td>( \frac{4\pi k \Delta T}{(1/r_1) - (1/r_2)} )</td>
</tr>
<tr>
<td><strong>Thermal resistance</strong> ((R_{t,cond}))</td>
<td>( \frac{L}{kA} )</td>
<td>( \frac{\ln (r_2/r_1)}{2\pi Lk} )</td>
<td>( \frac{(1/r_1) - (1/r_2)}{4\pi k} )</td>
</tr>
</tbody>
</table>

\(^a\)The critical radius of insulation is \( r_{cr} = k/h \) for the cylinder and \( r_{cr} = 2k/h \) for the sphere.
Next Topic

- Steady State Conduction
  - Thermal Energy Generation
Thermal Energy Generation

- Recall heat diffusion equation:

\[
\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + q = \rho C_p \frac{\partial T}{\partial t}
\]

- In many applications, other forms of energy such as chemical, nuclear, electrical etc. can be converted to thermal energy i.e. heat generation or heat source

- In other applications, thermal energy can be converted back to any other forms of energy i.e. heat destruction or heat sink
Thermal Energy Generation (contd.)

- Plane wall
- One-dimensional
- Steady state
- Constant heat source

\[ T(x) = \frac{qL^2}{2k} \left[ 1 - \left( \frac{x}{L} \right)^2 \right] - \frac{T_1 - T_2}{2} \frac{x}{L} + \frac{T_1 + T_2}{2} \]

- What are the differences as compared to plane wall with no heat source?
- Temperature distribution no longer linear
- Heat flux no longer constant
Thermal Energy Generation (contd.)

• Cylinder/Sphere
• One-dimensional
• Steady state
• Constant heat source

\[ T(r) = T_s + \frac{q r_o^2}{4k} \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right] \]

\[ T(r) = T_s + \frac{q r_o^2}{6k} \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right] \]

• See Appendix C for further analysis
• General analysis requires solving the heat equation with given BCs
Radioactive wastes are packed in thin-walled spherical container. The wastes generate thermal energy non-uniformly according to the relation \( q = q_o [1 - (r/r_o)^2] \), where \( q \) is the local rate of energy generation per unit volume, \( q_o \) is a constant and \( r_o \) is the radius of the container. Steady operating conditions are maintained by submerging the container in a liquid at \( T_\infty \) with convection coefficient \( h \).

Determine the temperature distribution, \( T(r) \), in terms of \( q_o, r_o, T_\infty, h \) and thermal conductivity of the radioactive wastes.
Example

- Consider uniform heat generation in a cylinder and in a sphere of equal radius made of the same material in the same environment. Which geometry will have a higher temperature at the center? Why?
HW # 2 will due on 10/23(Tuesday), right before the class!

Late HW will not be accepted!!
Closure

- Thermal energy generation in plane walls and radial systems for steady state, 1D heat conduction with constant properties
- Differences between temperature distributions, heat transfer rates and heat fluxes with and without thermal energy generation for plane walls and radial systems
- Thermal resistance network concepts are not applicable for cases with thermal energy generation or destruction because rate of heat transfer varies spatially
Next Topic

- Steady State Conduction
  - Extended Surfaces
    - General Conduction Analysis
    - Constant Cross-section Fins
    - Variable Cross-section Fins
How to keep cool?

**Brachiosaurus**
- No additional surface area; difficulty in maintaining body temperature

**Stegosaurus**
- Many extended surfaces; had a lower body temperature
Extended Surfaces

- Many applications involve heat exchange between a solid and surrounding fluid
  - convection
  - radiation (usually neglected or combined with convection)

\[ q_{\text{conv}} = hA_s (T_s - T_\infty) \]

- Convection heat transfer can be increased via:
  - increasing heat transfer coefficient
  - decreasing surrounding temperature
  - increasing available surface area
Extended Surfaces (contd.)

- Extended surfaces (fins) can be classified as:
  - Based on cross-section
    - Rectangular (straight fins and annular fins)
    - Circular (pin fins)
  - Based on heat transfer
    - Thin fins (one dimensional)
    - Thick fins (two dimensional)

- Selection of a particular fin type depends on several design considerations (space, weight, manufacturing, cost etc.)
Extended Surfaces (contd.)

**Figure 3.14** Fin configurations. (a) Straight fin of uniform cross section. (b) Straight fin of nonuniform cross section. (c) Annular fin. (d) Pin fin.
Extended Surfaces (contd.)

- Engine Fins
- Annular Fins
- Fin Arrays
- Serrated Fins
Extended Surfaces (contd.)

- **Surface**
- **Rectangular Fin**

- **Perimeter**, $P = (2W + 2t)$
- **Cross-sectional Area**, $A_c = Wt$
- **Surface Area**, $A_s = Px$

Diagram:
- **Fin Base**
- **Fin Tip**
- **Straight Rectangular Fin**
- **Surface**
- **Dimensions:** $L$, $W$, $t$, $x$
General Conduction Analysis

- Thin fin i.e. one-dimensional heat transfer in x direction
- Steady state
- Constant properties
- No thermal energy generation
- Negligible radiation

Differential Control Volume

\[ dq_{\text{conv}} \]

\[ q_x \]

\[ q_{x+dx} \]

\[ dx \]

\[ x \]
Applying conservation of energy for infinitesimally small differential element, we can write:

\[
\frac{d^2 T}{dx^2} + \left( \frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left( \frac{1}{A_c} \frac{h dA_s}{k dx} \right) (T - T_\infty) = 0
\]

The above differential equation governs temperature distribution within the fin for given assumptions.

In general, boundary conditions must be employed so as to obtain solution of the above differential equation.
Constant Cross-section Fins

- If the cross-sectional area remains constant, then the above differential equation can be written as:

\[
\frac{d^2 T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0
\]

- To simplify, we can write:

\[
\frac{d^2 \theta}{dx^2} - m^2 \theta = 0; \quad \theta = T - T_\infty \text{ and } m^2 = \frac{hP}{kA_c}
\]

- At the base of the fin, we typically specify temperature i.e.

\[
\theta_b = T_b - T_\infty
\]
At the tip of the fin, we typically specify four common boundary conditions:

- specify tip temperature
- assume insulated tip
- consider convection from tip
- model as infinitely long fin i.e. no temperature difference between tip and surrounding fluid
Table 3.4 summarizes the results for these boundary conditions for constant cross-section fins.

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip Condition ((x = L))</th>
<th>Temperature Distribution (\theta/\theta_b)</th>
<th>Fin Heat Transfer Rate (q_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Convection heat transfer: (h\theta(L) = -kd\theta/dx\big</td>
<td>_{x=L})</td>
<td>[\frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL}]</td>
</tr>
<tr>
<td>B</td>
<td>Adiabatic (d\theta/dx\big</td>
<td>_{x=L} = 0)</td>
<td>[\frac{\cosh m(L - x)}{\cosh mL}]</td>
</tr>
<tr>
<td>C</td>
<td>Prescribed temperature: (\theta(L) = \theta_L)</td>
<td>[\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L - x)}{\sinh mL}]</td>
<td>[M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}]</td>
</tr>
<tr>
<td>D</td>
<td>Infinite fin ((L \to \infty)): (\theta(L) = 0)</td>
<td>(e^{-\alpha x})</td>
<td>[M]</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\theta & = T - T_\infty \\
\theta_b & = \theta(0) = T_b - T_\infty \\
m^2 & = hP/kA_c \\
M & = \sqrt{hP}kA_c \theta_b
\end{align*}
\]
Example

Two large blocks of copper, each maintained at 400 K, are connected by a solid copper rod with a square cross section of 5 cm x 5 cm and a length of 50 cm. The rod is exposed to a forced flow of air at 300 K providing a heat transfer coefficient \( h = 100 \text{ W/m}^2\text{-K} \) along its length. Radiation heat transfer may be ignored. What is the temperature at the rod center? The following properties are given for copper: \( \rho = 8900 \text{ kg/m}^3 \), \( \text{Cp} = 385 \text{ J/kg-K} \), \( k = 400 \text{ W/m-K} \).
Example figure

Copper block at $T = 400 \text{ K}$

50 cm

Copper rod
5 cm x 5 cm square cross-section

Copper block at $T = 400 \text{ K}$
Pin fins are widely used in electronic systems. Consider a pin fin of uniform diameter $D$, length $L$ and thermal conductivity $k$ connecting two identical devices of length $L_g$ and surface area $A_g$. The devices are characterized by a uniform volumetric generation of thermal energy $q$ and thermal conductivity $k_g$. Assume that the exposed surfaces of the devices are at a uniform temperature corresponding to that of pin base, $T_b$. Heat is from exposed surfaces to an adjoining fluid with convection coefficient $h$. The back and sides of the devices are perfectly insulated.

Derive an expression for the base temperature $T_b$ in terms of the device parameters, the convection parameters and fin parameters.
A cylindrical shell of length $l$ (perpendicular to the page), diameter $D$ (much smaller than $l$), and thickness $b$ (much smaller than $D$) is made of a material having a thermal conductivity $k$. Half of the shell is embedded in an insulating wall while the exposed half is cooled by a fluid at $T_\infty$ having a convective heat transfer coefficient $h$. The coolant flows axially (perpendicular to the page) both inside and outside the shell. The embedded part of the shell generates heat at a uniform volumetric rate $q$. You may neglect any radiation effects.

Derive an expression for the steady-state temperature $T_1$ at the mid-plane of the shell as a function of the given parameters.

Derive an expression for the highest temperature, $T_{\text{max}}$, in the shell.
Copper tubing is joined to a solar collector plate of thickness $t$ and the working fluid maintains the temperature of the plate above the tubes at $T_o$. A uniform net radiation heat flux $q_{\text{rad}}$ exists at the top surface of the plate while the bottom surface is well insulated. The top surface is also exposed to a fluid at $T_\infty$ that provides a uniform convection with heat transfer coefficient $h$.

- Derive the differential equation governing the temperature distribution $T(x)$ in the plate
- Obtain a solution to the above differential equation for given boundary conditions
Copper tubing is joined to a solar collector plate as shown. The aluminum alloy (2024-T6) absorber late is 6 mm thick and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced a distance $L$ of 0.20 m from each other, and water is circulated through the tubes to remove the collected energy. The water may be assumed to be at a uniform temperature of $T_w=60 \, ^\circ C$. Under steady-state operating conditions for which the net radiation heat flux to the surface is $q_{\text{rad}}=800 \, \text{W/m}^2$.

What is the maximum temperature on the plate and the heat transfer rate per unit length of tube?
Note that $q''_{\text{rad}}$ represents the net effect of solar radiation absorption by the absorber plate and radiation exchange between the absorber and cover plate. You may assume the temperature of the absorber plate directly above a tube to be equal to that of the water.